Band degeneracy lifting

- Two e^- per state ↑ and ↓ based on Kramers degeneracy
  * space inversion symmetry: $E(\vec{k},\uparrow) = E(-\vec{k},\uparrow)$ (SIS)
  * Time reversal symmetry: $E(\vec{k},\uparrow) = E(-\vec{k},\downarrow)$ (TRS)

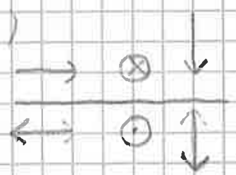
$\Longrightarrow E(\vec{k},\uparrow) = E(\vec{k},\downarrow)$

To lift: break TRS: magnetic order
break SIS: surface/interface crystal structure
(Rashba, Dresselhaus)

Symmetry operations on spin

Pseudo vector (bi-vector)

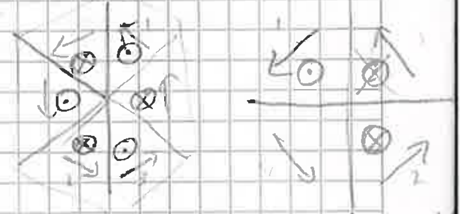
Mirror operation:



only $S_\perp$ at Mirror plane

TRS:
(point symmetry)

Combine



in-plane ✓
out-of-plane

(less obvious in real space)

☑ Allowed spin texture depends on symmetry ☑

① Break TRS: magnetic order

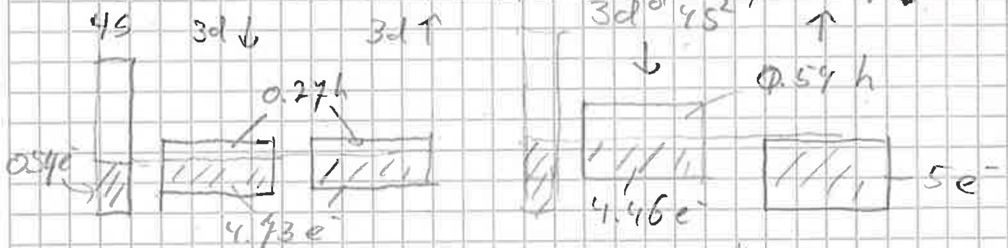
- Not dipole interactions ($T_c \sim 1\,K$)
- Exchange interactions: singlet vs triplet formation

Heisenberg $H^{spin} = -\sum J_{ij} \vec{S}_i \cdot \vec{S}_j$ $J > 0$ FM $J < AFM$

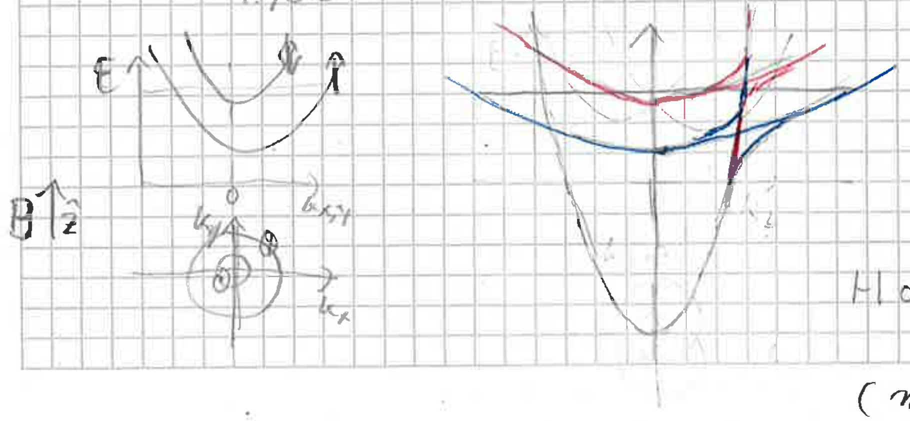
Ok for localised spin (insulators). No non half-integer μ_B
Not for itinerant e^-

Ferromagnet

- Stoner model: Ni as example ($\mu = 0.6$ per atom)



DOS +



Due to hybridization also sp-bands become spin polarised

How many mirror planes ??

(max 1)

No net M

Anti: ferromagnet: ↑ ↓ ↑ ↓ ↑ ↓

Many different AFM arrangements: layer, checkerboard etc.

"Hidden" order: cusps in specific heat and χ

Resolved by neutron diffraction

Bloch wave function for each sublattice

↳ translational symmetry ⟹ bands are degenerate

Altermagnet: similar to AFM but spin sublattices not connected by translation / inversion

rotation

other atoms

no net M

Charge: $\varphi = 2\pi$ spin: $\varphi = 4\pi$ (to same point)

⟹ degeneracy lifted?

Mirror planes?

many possible

At mirror plane: degeneracy enforced (without SOI)

(No TRS)

* generalise: spin helix etc. (non-colinear, non coplanar)
* Inclusion of SOI (generic)
* Micro structure ↳ Dzyaloshinskii-Moriya interaction

DMI $\mathcal{H}^{DMI} = \vec{D}_{ij} \cdot (\vec{S}_i \times \vec{S}_j)$

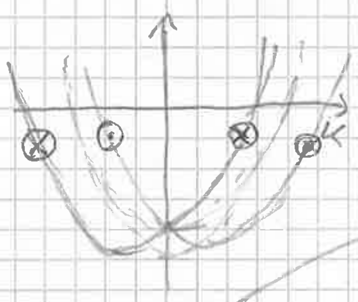
Canting of spin due to SOI

② Break SIS (keep TRS $\to \mathcal{E}(k=0,\uparrow) = \mathcal{E}(k=0,\downarrow)$)

* broken symmetry → potential gradient → $\vec{E}$-field
* Lorentz transformation force with $\vec{p} = \hbar \vec{k}$ in $\vec{E}$

$$B_z = \gamma\Big(\underset{\uparrow_0}{\vec{B}} - \frac{\vec{v}\times\vec{E}}{c^2}\Big)_z \implies \vec{B}_{eff} = \frac{1}{2m_e c}(\vec{p}\times\vec{E}) \text{ along } \hat{z}$$

$$\Delta\mathcal{E} = -\vec{\sigma}\cdot\vec{B}_{eff}$$ momentum dependent Zeeman splitting



Hamiltonian ($\vec{B}_{ext} = 0$)

$$\mathcal{H} = \frac{p^2}{2m_e} + U(r) - \frac{e\hbar}{(2m_e c)^2}\vec{\sigma}\cdot(\vec{E}(r)\times\vec{p})$$

$$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \quad \sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix} \quad \sigma_y = \begin{pmatrix}0&-i\\i&0\end{pmatrix}$$
$$\sigma_z = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$-\sigma\cdot(\vec{E}(r)\times\vec{p}) = \sigma\cdot(\nabla U(r)\times\vec{p}) = \frac{1}{r}\frac{\partial U}{\partial r}\sigma\cdot(\vec{r}\times\vec{p})$$

$$= \frac{1}{r}\frac{\partial U}{\partial r}\vec{\sigma}\cdot\vec{L}$$ spin-orbit interaction